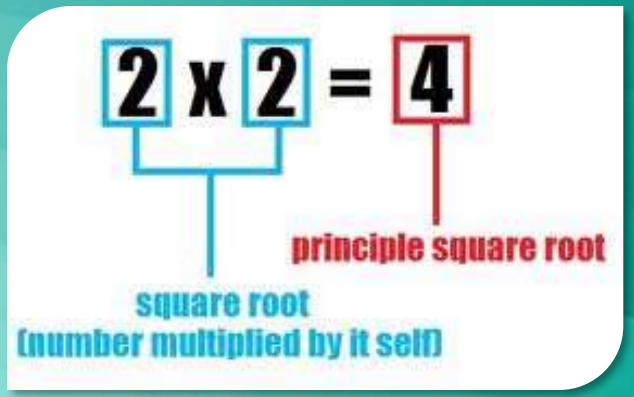
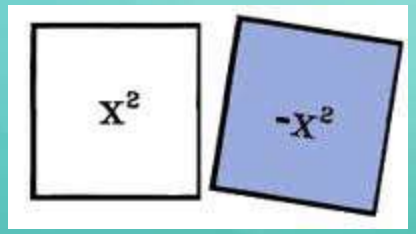
# Squares, Square Roots


$$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$$



**Have you ever wondered how far you can see out from an airplane or from the top of a hill? How far you can see depends on the curvature of Earth and your height above it.**

**You can use the formula $d = \sqrt{1.5h}$ to estimate the distance $d$ in miles to the horizon when $h$ is the height of the viewer's eyes above the ground in feet.**

**Suppose you are looking out a second floor window 25 ft above the ground. Find the distance you can see to the horizon. (Round to the nearest mile.)**

***Think about the relationship between the area of a square and the length of one of its sides.***

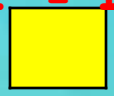
**Quilts are often pieced together from small squares to form a large design.**

4 feet

Area 16 sq ft

# Finding squares and square roots

$1^2 = 1$



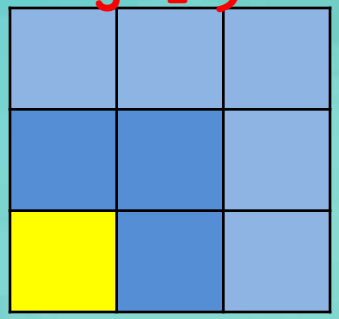
If the length of one side of a square is 2, then its area is _____?

$2^2 = 4$

If the area of a square is 4, then the length of one side is_____?

If the length of one side of a square is 3, then its area is_____ ?

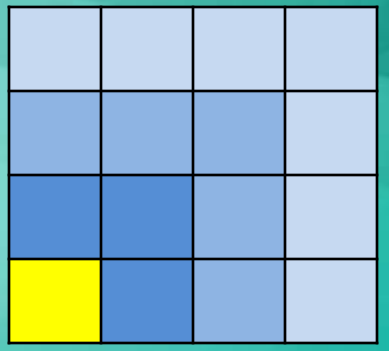
$3^2 = 9$



If the area of a square is 9, then the length of one side is____?

If the length of one side of a square is 4, then its area is_____?

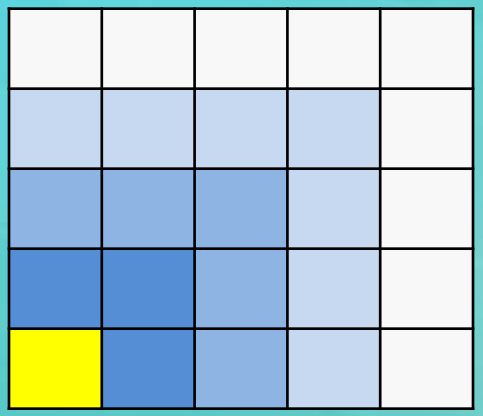
$4^2 = 16$



If the area of a square is 16, then the length of one side is_____?

The symbol for the ***principal***, or positive ***square root***, √ is called the ***radical sign***.

$5^2 = 25$



If the length of one side of a square is 5, then its area is_____?

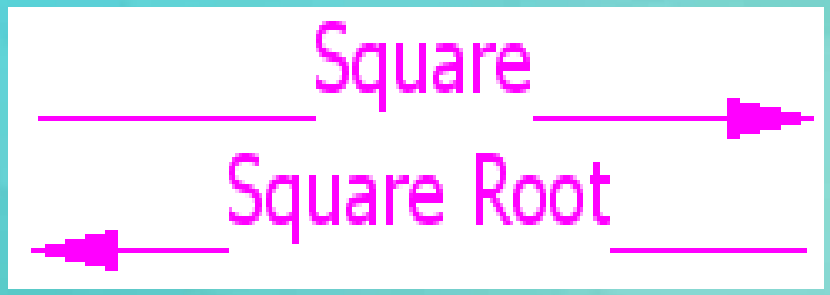
Given: area = 25
Length of side = √25 = 5

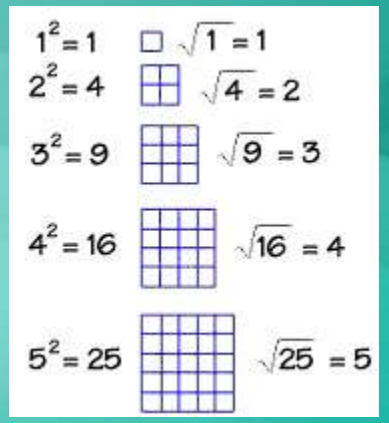
For any positive integer there are two square roots, one positive and one negative.

# Radical

a radical is a root (like a square root) of a number.
A radical is made up of a radical sign and
something inside called the radicand.

We discussed earlier that the inverse of an operation would “undo” that operation. The **inverse operation** of squaring a number is the square root ( √) of that number.



# You need to remember :

| Perfect Squares | Radicals (square roots) |
|---|---|
| $1 = 1 \times 1 = 1^2$ | $\sqrt{1} = 1$ |
| $4 = 2 \times 2 = 2^2$ | $\sqrt{4} = 2$ |
| $9 = 3 \times 3 = 3^2$ | $\sqrt{9} = 3$ |
| $16 = 4 \times 4 = 4^2$ | $\sqrt{16} = 4$ |
| $25 = 5 \times 5 = 5^2$ | $\sqrt{25} = 5$ |
| $36 = 6 \times 6 = 6^2$ | $\sqrt{36} = 6$ |
| $49 = 7 \times 7 = 7^2$ | $\sqrt{49} = 7$ |
| $64 = 8 \times 8 = 8^2$ | $\sqrt{64} = 8$ |
| $81 = 9 \times 9 = 9^2$ | $\sqrt{81} = 9$ |
| $100 = 10 \times 10 = 10^2$ | $\sqrt{100} = 10$ |

# Finding and Approximating Square Roots

**Find two consecutive integers between which √58 can be found.**

$7 \times 7 = 49$

too small

$8 \times 8 = 64$

too large

Thus, √58 is between 7 and 8. Using a calculator, √58 = 7.62

Find two consecutive integers between which

$-\sqrt{77}$ can be found.

Find two consecutive integers between which √35 can be found.

Digital pictures are made up of pixels (colored dots). The picture on the right is an enlargement of the picture on the left and shows the dots (pixels) more clearly. The ***square*** computer image contains 676 pixels. How many pixels tall is the icon?

Since the icon is ***square***, find the square root of 676 to find the length of the side.

$26^2 = 676$

so $\sqrt{676} = 26$.

The icon is 26 pixels tall.

In the order of operations, a square root symbol is like an exponent. Everything under the radical is treated as if it were in parentheses.

$$\sqrt{5-3} = \sqrt{(5-3)}$$

## Evaluate the expression

$$2\sqrt{16}+5$$

$$2\sqrt{16}+5=2(4)+5=8+5=13$$

## Evaluate the expression

$$\sqrt{9+16}+7$$

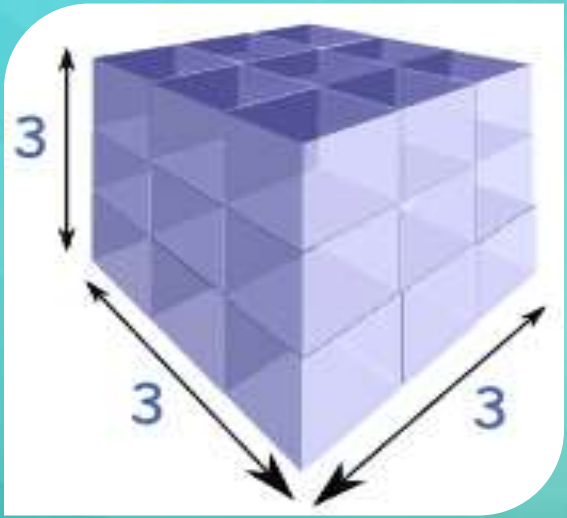
$$\sqrt{9+16}+7=\sqrt{25}+7=5+7=12$$

$\sqrt[3]{\ }$

$\sqrt[4]{\ }$

Cube roots, fourth roots and *n*th roots can also be found. These are easily done on the graphing calculator using the MATH key.

Cube root goes in the other direction, $3^3$ cubed is 27 so the ***cube root of 27 is 3.***




Notice the graphing calculator screen below left, under the MATH key you will find the cube root of a number. Here the ***inverse*** is the cube root of a number that is cubed, such as $3^3 = 27$, so the cube root of $27 = 3$

$$\sqrt[3]{27} = 3$$

# Words Problems Dealing with radicals

Squares and Square Roots

Cubes and Cube Roots

Ms. Estefan wants to put a fence around 3 sides of a square garden that has an area of 225 $ft^2$. How much fencing does she need?

Notice this problem tells you how many $ft^2$ there are in the garden and you must find the length of the sides.

Remember $A = S^2$, therefore

$225 = S^2$

$\sqrt{225} = 15$

you only need 3 sides of fencing, $15 \cdot 3 = 45ft$

## Try this one on your own.

A karate match is held on a square mat that has an area of 676 $ft^2$. What is the length of the mat?

**Measurement problems use square roots and cubed roots. Look as this problem.**

A cube has a volume of 1728 $cm^3$, what is the surface area of the cube?

In this problem you must find the cube root of the volume which is 12 cm. Then you must find the surface area of the cube.

$12 \cdot 12 \cdot 6 = 864\ cm^2$

## Word problems. Solve the following. (continued)

1) For high school wrestling competitions, the wrestling mat must be a square with an area of 1444 square feet. What is the length of each side of the wrestling mat?

2) A square picture frame measures 36 inches on each side. The actual wood trim is 2 inches wide. The photograph in the frame is surrounded by a bronze mat that measures 5 inches wide. What is the maximum area of the photography?

## Word problems continued.

3) A box of tile contains 12 tiles. If you tile a square area using whole tiles, how many tiles will you have left?

4) A can of paint claims that one can will cover 400 square feet. If you painted a square with the can of paint, how long would it be on each side?

# Simplifying Radicals

To *simplify* means to find another expression with the same value. It does not mean to find a decimal approximation.

## To simplify (or reduce) a radical:

1) Find the largest perfect square which will divide evenly into the number under the radical sign. This means that when you divide, you get no remainders, no decimals, no fractions.

Reduce: $\sqrt{48}$ the largest perfect square that divides evenly into 48 is 16.

2) Write the number appearing under the radical as the product (multiplication) of the perfect square and your answer from dividing.

$\sqrt{48} = \sqrt{16} \cdot 3$

3) Give each number in the product its own radical sign.

$\sqrt{48} = \sqrt{16} \cdot 3 = \sqrt{16} \cdot \sqrt{3}$

4) Reduce the "perfect" radical which you have now created.

$\sqrt{48} = \sqrt{16} \cdot 3 = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

5) You now have your answer.

$\sqrt{48} = 4\sqrt{3}$

## What happens if I do not choose the largest perfect square to start the process?

If instead of choosing 16 as the largest perfect square to start the process, you choose 4, look what happens…

$$\sqrt{48} = \sqrt{4} \cdot 12$$

$$\sqrt{48} = \sqrt{4} \cdot 12 = \sqrt{4} \cdot \sqrt{12} = 2\sqrt{12}$$

Unfortunately, this answer is not in simplest form.

The 12 can also be divided by a perfect square (4).

$$2\sqrt{12} = 2\sqrt{4} \cdot 3 = 2\sqrt{4} \cdot \sqrt{3} = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$$

If you do not choose the largest perfect square to start the process, you will have to repeat the process.

## Example :

Reduce: $3\sqrt{50}$

Don't let the number in front of the radical distract you. It is just "along for the ride" and will be multiplied times our final answer.

The largest perfect square dividing evenly into 50 is 25.

$$3\sqrt{50} = 3\sqrt{25} \cdot 2 = 3\sqrt{25}\sqrt{2}$$

Reduce the "perfect" radical and multiply times the 3 (who is "along for the ride")

$$3\sqrt{25}\sqrt{2} = 3 \cdot 5\sqrt{2} = 15\sqrt{2}$$

# The End

## Thanks